

Tensile Membrane Action and the Fire Resistance of Steel Framed Buildings

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ABSTRACT

Tensile membrane action is a load carrying mechanism in reinforced concrete floors at very large deflection. Under this action, the load carried by the reinforced concrete floor can be many times higher than the design load carrying capacity determined at small deflections.

Recent full scale fire tests and fire accidents in steel-framed buildings with reinforced concrete floors have identified tensile membrane action as the main load carrying mechanism. This mechanism is able to transfer the applied loads on the floor to the supporting columns when the supporting steel beams to the floor are damaged and have lost their load carrying capacities.

The understanding of tensile membrane action in reinforced concrete floors has a significant implication for the fire protection of steel-framed buildings with reinforced concrete floors. This is because the supporting steel beams may be allowed to fail without endangering the safety of the whole building. Consequently, fire protection to these steel beams can be reduced or eliminated.

This paper describes in some detail an analytical procedure developed by the author for predicting tensile membrane action in reinforced concrete floors. An example is given which shows how to use this method to determine the fire resistance of a steel-framed building.

KEY WORDS: tensile membrane action, reinforced concrete slabs, fire resistance, fire protection, steel-framed buildings.

INTRODUCTION

Traditional studies on the structural behaviour of a building have mostly concentrated on the

behaviour of individual members of the building. This process of investigation has led to the development of design codes and standards worldwide based on checking the adequacy of individual members [1,2].

Structural design for fire is also based on the behaviour of individual elements. Although the design process is changing from the traditional prescriptive rules of fire resistance and fire rating [3] to the more rational performance related codes and standards [4,5], these more advanced codes and standards are still based on the behaviour of individual members. Consequently, the current practice is to prevent the failure of individual structural members. Although this practice has been proven to be adequate, it places onerous constraint on the construction of the building, often demanding excessive fire protection for all structural members.

The current requirement for the fire protection of structural members is especially severe for steel framed structures because the temperatures in exposed steel structural members increase rapidly and this results in a sharp reduction in their stiffness and strengths. However, there are many built-in redundancies in a building and the inherent resistance of a building is often much higher than that of its weakest components. The failure of a few loadbearing structural members in the building under fire conditions is not automatically followed by the collapse of the building. When some loadbearing members in the building do fail, the building itself often has the ability to bridge over the damaged parts. This phenomenon has been observed in fire damaged buildings [6] and was recently demonstrated by a full-scale fire test conducted by the Building Research Establishment [7].

The ability of a reinforced concrete slab to bridge over damaged loadbearing steel beams is the result of tensile membrane action. Under tensile membrane action, the floor slabs of the building can withstand loads many times higher than the design strength of the floor at small deflections. The ability of the building to develop tensile membrane action and to bridge over damaged parts has significant implication for the fire resistance and fire protection of steel-framed buildings with reinforced concrete floors. Relying on tensile membrane action to resist the applied loads after the loss of some of the supporting steel beams implies that these steel beams may be allowed to fail under fire conditions. This implies that these steel beams supporting reinforced concrete slabs may not need fire protection.

It became apparent that to better understand the behaviour of whole buildings and explore the potential of using unprotected steel beams under fire conditions, the effect of tensile membrane action in the floor slab should be thoroughly investigated.

Tensile membrane action in reinforced concrete slabs was a popular topic in the 1960's and 1970's. However, the development of theories on this subject was limited, mainly due to the unforeseen beneficial effects of tensile membrane action on the behaviour and design of buildings under fire conditions. The more significant advancement in the understanding of tensile membrane action are the contributions of Park [8], Wood [9] and Kemp [10].

Park [8] studied tensile membrane action in reinforced rectangular slabs with clamped edges. Wood [9] developed a unique solution for simply supported circular plates under tensile membrane action. Kemp [10] obtained the solution for the load-deflection response of a simply

supported square slab. Further development in this area is limited to extending the approach of Kemp [10] to simply supported rectangular slabs [11]. However, there has been no study on tensile membrane action in reinforced concrete slabs with more complex support conditions. For example, there is no method for predicting the behaviour of rectangular slabs with partial strength edge supports or rectangular slabs with simple edge supports plus interior columns.

In this paper, a methodology is developed to determine the load-deflection relationships of reinforced concrete slabs under tensile membrane action at large deflections. This method can be applied to reinforced concrete slabs with various supporting conditions. An example is then given to show how this method may be applied to determine the fire resistance and fire protection of a steel framed building.

GENERAL BEHAVIOUR

Figure 1 illustrates schematically the load-deflection curves of reinforced concrete slabs with clamped and simply supported edges.

Calculating the load carrying capacity of a reinforced concrete slab for ambient temperature design is generally based on the slab failing in pure flexural bending. This represents the first failure mode of the slab at small deflections. Johansen's [12] yield line theory is often thought to give an upper bound (points A in figure 1) to the load carrying capacity of the slab and this approach has been adopted in many codes of practice for the design of reinforced concrete slabs.

However, if the slab is laterally restrained, the slab will arch from boundary to boundary. This results in the development of compressive membrane forces in the concrete slab, increasing its load carrying capacity. Tests on buildings [13] and laterally restrained individual slabs [9] confirmed the enhanced load carrying capacities of concrete slabs due to compressive membrane action. The load carrying capacity of the slab at maximum compressive force (point B in figure 1) may be several times the strength according to yield line theory.

As the slab deforms further, the depth of the cracked concrete increases and the available uncracked concrete for compressive stress diminishes. The membrane action in the slab changes from compressive to tensile. When cracking extends over the entire depth of the concrete cross-section (point C in figure 1), the applied load on the reinforced concrete slab can be regarded as being taken by tensile membrane action in the steel reinforcement. During this stage of loading, the applied load on the slab is supported by the net of reinforcement anchored on the supporting edges. The load carrying capacity of the slab increases with increasing slab deflection. The slab collapses when the slab reinforcement fractures at point D in figure 1.

For a simply supported slab, its ability to develop compressive membrane action is very limited. However, at large deflection, a slab can develop an in-plane ring beam in compression to support the development of tensile membrane action in the central region of the slab. This behaviour was confirmed by Brotchie and Holley's tests [14]. For a simply supported slab, there is a smooth transition from pure flexural behaviour at small deflection to tensile membrane action at large deflection as shown in figure 1.

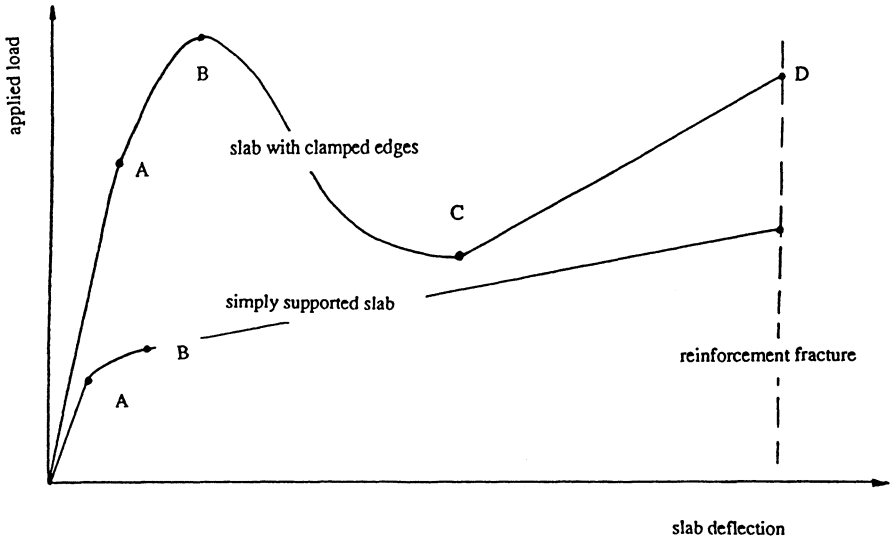


Figure 1: Complete load-deflection curve for a reinforced concrete slab

Figure 1 clearly shows that for a reinforced concrete slab with clamped edges, the behaviour of compressive membrane action is unstable. Furthermore, compressive membrane behaviour is extremely sensitive to edge restraints and initial imperfections. The realistic supporting conditions for reinforced concrete slabs in real buildings are seldom completely restrained. These are the obstacle to the wider acceptance of compressive membrane action by designers.

Tensile membrane action is stable, but it occurs at very large deflections in reinforced concrete slabs. For normal design at cold condition, these large deflections would violate serviceability conditions. However, deflection ceases to be a problem when the slab is subjected to an accidental loading such as fires. It is therefore acceptable to explore the enhanced load carrying capacity of the reinforced concrete slab due to tensile membrane action at large deflections to assess its collapse strength for fire safety design.

TENSILE MEMBRANE ACTION IN RECTANGULAR SLABS

Explicit solution for simply supported slabs

The explicit determination of the load-deflection relationship for a reinforced concrete slab

under tensile membrane action involves the selection of the yield line pattern in the slab, the specification of the yield condition, the consideration of geometrical compatibility and force equilibrium and the application of the work method [9].

Following the approach of Wood [9] for a simply supported circular slab, Kemp [10] obtained the solution for the load-deflection behaviour under tensile membrane action for a simply supported square slab with isotropic reinforcement. His analysis adopted a diagonal yield line mechanism for the square plate. This was justified on the ground of symmetry.

Under tensile membrane action at large deflections in the slab, Kemp [10] gave the following equation to describe the slab load-maximum deflection relationship:

$$\frac{P}{P_y} = 1 + \beta' \left(\sqrt{\frac{\delta_{\max} F}{2\beta' M_p}} - 1 \right)^2 \quad (1a)$$

where

- P is the applied load on the slab
- P_y the slab load carrying capacity according to yield line analysis
- F the tensile force in reinforcement per unit width of slab,
- M_p the slab bending moment capacity per unit width,
- δ_{\max} maximum deflection in the slab and
- β' function of the reinforcement ratio of the slab, given by

$$\beta' = \frac{\frac{1}{2} \frac{F}{\sigma_b d}}{1 - \frac{1}{2} \frac{F}{\sigma_b d}} \quad (1b)$$

where d is the effective depth of the slab and σ_b is the bending strength of the uniform concrete stress block. $\sigma_b = 0.67\sigma_{cu}$ according to BS 8110 [15] in which σ_{cu} is the concrete cube strength. The concrete cylinder strength σ_{cc} can be determined from the following relationship: $\sigma_b \approx 0.83\sigma_{cc}$ which is taken from EC 4 [16].

Equations (1a) and (1b) are directly applicable to rectangular slabs if diagonal yield line pattern is assumed.

General solution

Although the method of Wood [9] and Kemp [10] may be applied to slabs with more complicated yield line patterns, the derivation would become extremely complicated because of the increase in the number of concrete blocks bound by yield lines. However, the load-deflection curve of a simply supported rectangular slab with diagonal yield lines may be used in conjunction with the procedure described in the following section to give the general solution for the load-deflection behaviour of the rectangular slab with more complicated support

conditions, for example, simply supported edges plus interior column supports.

The starting point is the work method, which is generally expressed in the following form:

$$\delta W_{ext} = \delta W_{int} \quad (2)$$

The increment in work done by external forces δW_{ext} is:

$$\delta W_{ext} = P \cdot \Delta \delta_{max} U_p \quad (3)$$

where $\Delta \delta_{max}$ is the increment in the maximum slab deflection, P is the external applied load and U_p is the unit work done by the unit external force over a unit increment in the maximum slab deflection.

The increment in work done by internal forces δW_{int} is the sum of the work done by plastic moments along the yield lines and reinforcement forces.

$$\delta W_{int} = M_p \Delta \delta_{max} U_m + F \cdot \Delta \delta_{max} U_f \quad (4)$$

where M_p is the plastic moment capacity of a unit width of the slab, F is the reinforcement yield force of a unit slab width and U_m is the unit work done by the plastic moments over a unit increment in the maximum slab deflection over the whole slab. U_f is the derivative of the work done by the unit reinforcement force over the whole slab with regard to the maximum slab deflection $\Delta \delta_{max}$.

The work done by the reinforcement forces is the product of the extension in the reinforcement and the unit reinforcement force over the whole slab. If the slab is clamped in plane along its supporting edges, the reinforcement extension is the difference between the length of the deformed profile of the slab strip containing the reinforcement and the original reinforcement length.

If the slab is not clamped as in real buildings, this assumption is not strictly true. However, tests on simply supported slabs by Brotchie and Holley [14] support the view that a compressive ring beam will form within the slab to support the tensile forces in the reinforcement in the centre of the slab.

Substituting equations (4) and (3) into equation (2), the load carrying capacity of the slab under tensile membrane action is:

$$P = M_p U_m / U_p + F U_f / U_p \quad (5)$$

Since the first term on the right hand side of equation (5) is the yield line solution (P_y) of the slab, equation (5) becomes:

$$P - P_y = F U_f / U_p \quad (6)$$

Since the extension of the reinforcement is a quadratic function of the maximum slab deflection δ_{\max} , the derivative U_f may be more conveniently written as:

$$U_f = U_{fu} \delta_{\max} \quad (7)$$

Substituting equation (7) into equation (6) gives the load-deflection relationship of the slab under tensile membrane action as:

$$P - P_y = \frac{FU_{fu}}{U_p} \delta_{\max} \quad (8)$$

The values of U_m , U_p , U_{fu} and P_y depend on the deformed shape and geometry of the slab only. Therefore, once the yield line pattern is determined, these values can be easily obtained.

As mentioned earlier, equation (8) may not be accurate for slabs with boundary conditions other than clamped edges when the calculation of U_{fu} is based on the clamped edges. The result of assuming clamped edges is to increase the deformation in the reinforcement and this results in an overestimation in the load carrying capacity of the slab. However, the error of adopting this assumption would be greatly reduced if the influence of this assumption on the slab load-deflection behaviour is similar for different deformation patterns of the slab. In other words, equation (8) may be modified by a coefficient to account for the influence of boundary conditions as:

$$P - P_y = k_b \frac{FU_{fu}}{U_p} \delta_{\max} \quad (9)$$

in which k_b is 1 for slabs with clamped edges. For other edge restraint conditions, k_b is lower than 1. If k_b is a constant for a slab irrespective of its deformed shape, the ratio of equation (9) for any two systems may be used to eliminate k_b . The relationship between these two systems may be expressed as:

$$\frac{(P - P_y)_1}{(P - P_y)_2} = \frac{(U_{fu} / U_p)_1}{(U_{fu} / U_p)_2} \quad (10)$$

where subscripts 1 and 2 refer to deformation patterns 1 and 2 respectively.

If the exact solution for the slab load-deflection relationship under tensile membrane action with one deformation pattern, which may not be correct, is determined, the solution of the slab with the more realistic deformation pattern can be found using equation (10).

The exact load-deflection relationship for the slab with diagonal yield line pattern is given in equation (1a). Once the correct yield-line pattern is identified for the slab, equation (10) can be

combined with equation (1a) to give the correct load-deflection relationship. Since equation (1a) is expressed as the ratio of the applied load P to the yield line solution P_y , equation (10) can be cast in the following different form:

$$\frac{(P-P_y)/P_y)_1}{((P-P_y)/P_y)_2} = \frac{(U_{fu}/U_m)_1}{(U_{fu}/U_m)_2} \quad (11)$$

When applying equations (1) to (11) to reinforced concrete slabs under fire conditions, the values of high temperature strength for both reinforcement and concrete are used.

VALIDATION

Due to the lack of experimental data, the proposed method cannot yet be extensively validated. Nevertheless, the following example may be used to illustrate the accuracy of the method.

Comparison with a multi-panel flat slab test

More than thirty years ago, a series of tests on concrete slabs were conducted at the University of Illinois, Urbana [17]. Hatcher, Sozen and Siess reported a selection of tests [18]. Each structure comprised nine 1524 mm square panels arranged three by three. The edges of the slab were supported by beams and 4 interior columns at one-third points of the slab were used to support the slab internally.

Ref. 18 includes one test (Test 138) whose slab was loaded to failure. The mode of failure was shear, by punching through the slab at an interior column. The failure load was calculated using the yield line theory to be at 0.0153 N/mm². However, this upper bound load was lower than the test failure load of 0.0173 N/mm². Since no strain hardening of the reinforcement was observed, this increase in the failure load was attributed to tensile membrane action in the slab. Although the increase in the failure load is not very significant, a comparison between the prediction of equation (11) and the test result is made to give a limited validation.

The yield line pattern shown in figure 2 was used in ref. 18 and is adopted in this paper. According to this yield line pattern, the value of U_{fu} , U_p and U_w can be calculated as: $U_{fu}=32$, $U_p=4L^2$ and $U_m=80$, where $L=1524$ mm.

Assuming equal positive and negative slab reinforcement, the unit moment capacity in the slab may be calculated as $0.0153*U_p/U_m = 1780$ N mm/mm. The thickness of the slab was 4.45 mm. Assuming an effective depth of $0.9*4.45=4$ mm, the equivalent unit membrane force in both the positive and the negative reinforcement can be calculated as $1780/4=445$ N/mm, giving a total membrane force of 890 N/mm.

The maximum deflection of the slab at failure was about 17.78 mm (=0.012*span of 1524 mm). Equation (9) gives $P=0.0207$ N/mm² at $k_6=1$. Clearly, this value is much higher than the observed failure load of 0.0173 N/mm².

+ column position - - - - - negative yield line ——— positive yield line

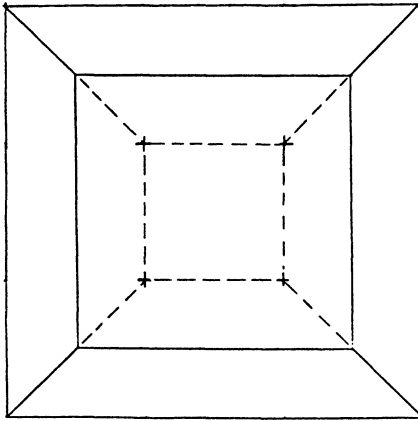


Figure 2: Yield lines for the multi-panel flat slab

The value of concrete cylinder strength was reported to be 18.5 N/mm^2 in ref. 18. Substituting the relevant values into equation (1b) gives $\beta^7=0.0376$. At the maximum slab deflection of 17.78 mm , equation (1a) gives $(P-P_y)/P_y=0.077$ corresponding to the diagonal yield line pattern. Under this slab deformation pattern, the values of U_{fu} and U_m can be calculated as: $U_{fu}=4$ and $U_m=16$. Using equation (11) gives $(P-P_y)/P_y=1.6*0.077=0.123$ for the more realistic yield line pattern of figure 4. The failure load predicted by equation (11) is therefore 0.0172 N/mm^2 . This value is much closer to the test failure load of 0.0173 N/mm^2 .

AN EXAMPLE

Figure 3(a) shows the floor plan of one compartment in the steel-framed multi-storey office building erected in the Building Research Establishment's Cardington Large Building Test Facility (LBTF). The floor comprises an in-situ concrete slab acting compositely with a steel decking of 0.7 mm thick supported by steel beams of. A layer of anti-cracking mesh of $142 \text{ mm}^2/\text{m}$ was placed 50 mm below the concrete surface. The steel decking spans as indicated in figure 3(a). The effective concrete thickness is 70 mm for composite action with the internal steel beams. The steel yield stress is 300 N/mm^2 and the concrete compressive strength is 35 N/mm^2 . The yield stress of the anti-cracking mesh is 600 N/mm^2 . Assuming an effective concrete width of 1.5 m for the internal steel beam, the plastic moment capacity of the composite beam is 543.2 kN.m .

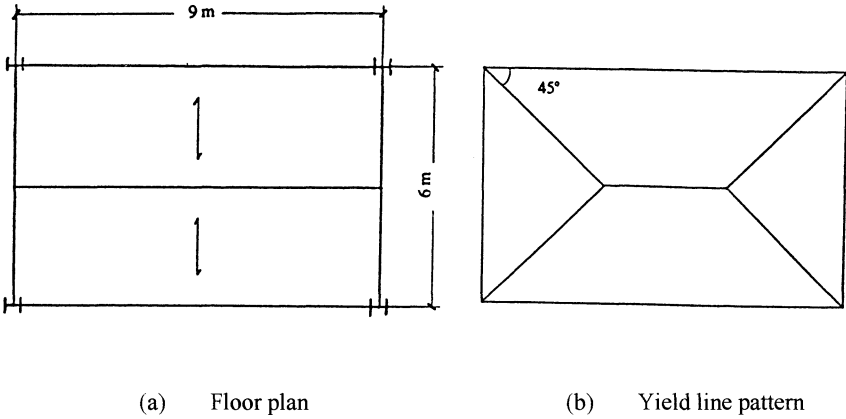


Figure 3: BRE fire test floor

The self-weight of the floor slab is 2.5 kN/m^2 and the imposed load on the slab is 2.4 kN/m^2 . The load density on the steel beam is therefore $3 \times (2.5 + 2.4) = 14.7 \text{ kN/m}$. This gives a bending moment in the beam of 148.8 kN.m (assuming simply supported boundaries). The load ratio is $148.8 / 543.2 = 0.274$. According to Eurocode 3 [5], the limiting temperature of the steel beam is $678 \text{ }^\circ\text{C}$, i.e. the steel beam will fail if its temperature under fire conditions is high than this value.

If steel is unprotected under fire conditions, its temperature increase follows that of the fire very closely. Temperatures in fires normally reach $1000 \text{ }^\circ\text{C}$ and the temperature in the unprotected steel is usually very close to this value. At a failure temperature of only $678 \text{ }^\circ\text{C}$, the steel beam would have to be protected according to Eurocode 3 [5].

However, the floor slab has sufficient strength under tensile membrane action. Assuming both the steel decking and the internal steel beam are unprotected and lose their load carrying capacities under fire conditions, the only reinforcement in the slab will be the anti-cracking mesh. The slab will have a span of 6 m instead of 3 m . According to flexural bending theory, the slab will not be able to sustain the applied load of 4.9 kN/m^2 .

Due to the insulation effect of concrete, temperatures at the unexposed slab surface and in the reinforcement are low that both concrete and reinforcement retain their strength. For the slab, $F = 85.2 \text{ N/mm}$ and $M_p = 4105 \text{ N.mm/mm}$. According to equation (1b), $\beta^* = 0.0379$. Equation (1a) gives the slab load-deflection relationship for the diagonal yield-line pattern as:

$$\frac{P - P_y}{P_y} = 0.0365 \left(\sqrt{\frac{\delta_{\max}}{3.519}} - 1 \right)^2 \quad (12)$$

Also for the diagonal yield line pattern, $U_{fu} / U_m = 0.5$. For the more realistic yield line pattern shown in figure 3(b), $U_m = 10$, $U_{fu} = 6$ and $U_p = 21 \text{ m}^2$. This gives $P_y = 1.95 \text{ kN/m}^2$. Substituting equation (12) into equation (11) gives the load-deflection curve for the more realistic yield line pattern of 5(b) as:

$$\frac{P - P_y}{P_y} = 0.0438 \left(\sqrt{\frac{\delta_{\max}}{3.519}} - 1 \right)^2 \quad (13)$$

At the applied load of 4.9 kN/m^2 , the deflection according to equation (13) is 166 mm . Under tensile membrane action, the collapse of the slab is characterized by the fracture of reinforcement at a deflection of about 10% of the shorter span of the slab according to Park [8]. The slab deflection of 166 mm is much lower than this limit. Therefore, the slab will be stable under fire conditions.

In fact, the fire test on this floor has already been carried out [7] with unprotected steel beams. The fire load was 40 kg wood/m^2 with a ventilation area of 37.8 m^2 . The temperature in the steel beam was more than $900 \text{ }^\circ\text{C}$. The steel decking de-bonded in many places and was badly distorted. However, the floor slab did not collapse. Moreover, the final maximum deflection of the slab after cooling down was about 160 mm , indicating that the theory is quite accurate.

CONCLUSIONS

In this paper, a method is developed to determine the load-deflection response of a rectangular reinforced concrete floor slab under membrane action.

This method has been shown to give very good estimates of the ultimate load carrying capacity of a multi-panel flat slab.

An example is given to show how the load-deflection of a reinforced concrete slab under membrane action may be determined and how this load-deflection relationship may be used to determine the fire resistance of a steel-framed office building. In this example, the load-deflection relationship of a fire test floor with a damaged steel beam and decking was calculated. The calculated deflection was very close to the test result. Moreover, this example demonstrated that the floor slab has the ability to bridge over the damaged steel beam under fire conditions. Since the floor slab has sufficient load carrying capacity, the steel beam needs no fire protection.

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