Mathematical Modeling of the Self-Heating Behaviour of Dusts around a Power Cable

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ABSTRACT

This paper has established mathematical models which describe the steady-state heat transfer in the self-heating process of dusts around a working power cable. The self-heating behaviour of the dusts was discussed by the numerical solutions of the established models associated with the tested self-heating parameters. Critical spontaneous igniting conditions for the dusts with and without inner heat sources were compared using typical coal dusts. The results show that the self-heating dusts around the power cable are very easy to ignite spontaneously. The small working power cable acted as a weak inner heat source of the dusts around the cable plays an important role in the self-heating of the dusts. Inversely, the spontaneous ignition of the dusts around the cable may cause a power cable fire.

KEYWORDS: spontaneous ignition, dusts self-heating, mathematical model, numerical solutions, critical spontaneous ignition temperature/radius

NOMENCLATURE

A	pre-exponential factor, $mol/m^3 \cdot s$
E	activation energy (J/mol)
Gr	Grashof number, $Gr = \beta g \Delta T d^3 / \nu_a^2$
Pr	Prantl number, $Pr = C_{p,a} \mu_a / \lambda_a$
Q	coal combustion heat, $3 \times 10^5 J/molO_2$
Q_d	natural convection heat exchange, W
q_0	heat generation per volume, W/m^3
r*	characteristic dimension of sample, mm
T_C	uniform temperature within the power cable metal, K
T	dust temperature, K
$\bar{T}$	characteristic temperature of air film, K
T_k	critical self-ignition temperature, K
T_S	temperature on the dust surface, K

Q_r heat generated by cable and dusts, W
r the radius of dust, m
r_1' critical ignition radius by F-K model, m
r_0 the radius of power cable metal, m
0.69×10^{-3} m for the XV power cable
T_∞ ambient air temperature, K
α heat transfer coefficient, W/m^2K
λ thermal conductivity, 0.116 W/mK
δ_C critical F-K number, 2.0 for a cylinder
ρ density of coal sample, 760 kg/m^3

1. INTRODUCTION

The settled dusts in the trench of the power cable often result in power cable fires in such dusty surroundings as mining well, power and coal chemical plants. For instance, the self-heating of bulk coal dusts in the power cable trench gave rise to a cable fire at Lanzhou petrochemical plant in 1993, which caused a tremendous loss.

The spontaneous ignition occurs because self-heating dusts react with oxygen in air and the exothermic oxidation takes place even at initially ambient conditions. It is well known that if the heat released from the oxidation is not dissipated rapidly enough to the surroundings, the oxidizing reaction will accelerate automatically. In these circumstances, an accumulation of the dusts may undergo a hazardous thermal runaway. Recently, many studies on the self-heating of dusts without any inner heat sources have been done, but the research on dusts with an inner heat source is quite limited [1-4].

A working power cable will accelerate the self-heating of the dusts on its surface. Therefore, the self-heating dusts around the cable is subject to inducing power cable fires.

2. MATHEMATICAL MODELS

2.1 The Basic Assumptions for Establishing Mathematical Models

When a power cable acts as the inner heat source of dusts on its surface, a complicated self-heating process occurs.

In order to establish a mathematical model for the steady-state heat transfer of the self-heating process, the following assumptions (cf. Figure 1) are made:

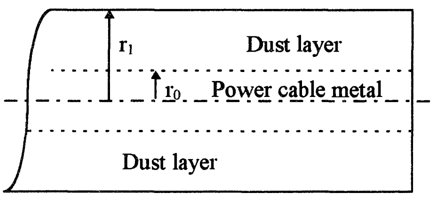

FIGURE 1 Power line and dusts on its surface

1) In comparison with the thickness of the dusts, the cable is infinite long, the heat conducting along the radial direction is one-dimensional.

2) The dust-layer is even, the shape of the dusts is a infinite long cylindrical column.

3) The heat generation per unit volume of the cable metal is a constant, i.e., q_0.

4) The dust-layer transfers its heat to ambient air by convection; the critical ignition condition for the dusts with an inner heat source is that the rate of heat release from the cable and the dusts equals to the rate of convected heat.

2.2 Theoretical Analyses on Heat Conduction

<u>**2.2.1 The heat conduction through power cable metal**</u> The heat conduction through the power-cable metal is negligible because the metal conductivity is quite great. Therefore, the temperature within the metal is uniform (T_C), which can be determined together with differential equations for the heat conduction through dust-layer, correspondent boundary conditions and heat balance equations. This will be discussed in this paper later.

<u>**2.2.2 Differential equations for the heat conduction through dust-layer**</u> If L stands for the axial length along the cable, the heat balance within the cylindrical volume element of dusts, from radius r to r+dr (cf. Figure 2), can be expressed as,

$$\begin{Bmatrix}\text{the heat conducted to}\\ \text{the volume element}\end{Bmatrix}+\begin{Bmatrix}\text{the heat generated in}\\ \text{the volume element}\end{Bmatrix}=\begin{Bmatrix}\text{the heat conducted out}\\ \text{of the volume element}\end{Bmatrix} \quad (1)$$

where, the heat conducted to the volume element:

$$-\lambda \cdot 2\pi r L \cdot \frac{dT}{dr};$$

the heat generated within the volume element:

$$2\pi r L dr \cdot Q \cdot A \cdot \exp\left(-\frac{E}{RT}\right);$$

the heat conducted out of the volume element:

$$-\lambda \cdot 2\pi(r+dr)L \cdot \frac{d}{dr}\left(T + \frac{dT}{dr}dr\right)$$

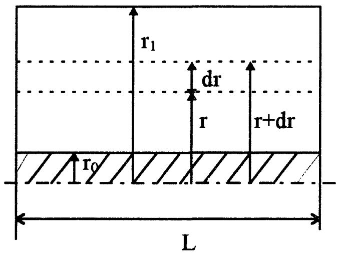

FIGURE 2 The cylindrical volume element of dusts around the power cable

Taking the specific terms into Eqn. (1) and neglecting the higher infinitesimal, the equation describing the temperature distributions within the dust can be written as:

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{QA}{\lambda}\exp\left(-\frac{E}{RT}\right) = 0, \quad (r_0 < r < r_1) \quad (2)$$

There is no analytic solution to Eqn. (2) but approximate numerical solutions exist.

<u>**2.2.3 Natural convection**</u> Here, α stands for natural convection coefficient, T_∞ for ambient air temperature, and $\bar{T} = \frac{1}{2}(T_s + T_\infty)$ for characteristic temperature of air film[5].

When the natural convection heat exchange between the surface dusts and air equals to the heat generated by the power cable and the self-heating dusts, a steady-state heat transfer process will be established, saying,

$$Q_d = \alpha \cdot 2\pi r_1 \cdot L(\bar{T} - T_\infty) \quad (3)$$

$$Q_r = \int_{r_0}^{r_1} 2\pi r L Q A \exp\left(-\frac{E}{RT}\right)dr + \pi r_0^2 L q_0 \tag{4}$$

$$\int_{r_0}^{r_1} 2 r Q A \exp\left(-\frac{E}{RT}\right)dr + r_0^2 q_0 = \alpha r_1 (T_s - T_\infty) \tag{5}$$

Eqn. (5) determines the critical spontaneous ignition condition of dusts around the power cable. Namely, under a certain ambient temperature (T_∞), if the cable radius (r_0) and dust radius (r_1) are given, the critical spontaneous ignition temperature (T_S) on the dust surface can be obtained; or if the cable radius (r_0) and the temperature on the dust surface (T_S), are known, the critical spontaneous ignition radius (r_1) of dusts can be determined.

2.3 Determination of the Relevant Parameters

2.3.1 Heat generation of power cable metal   A frequently used XV power cable is selected to show the effect of a weak inner heat source on the dust self-heating process. The heat generation per unit volume of cable metal, q_0, is calculated to be 2.6×10^6 $W\cdot m^{-3}$ using the relevant parameters of XV-type power cable.

2.3.2 Self-heating of coal dusts   For a bituminous coal under forced convection condition, its oxidizing reaction will be zero order related to the oxygen concentration. In these circumstances, the Frank-Kamenetskii model, which describes the relationship between sample dimension and its correspondent critical temperature of spontaneous ignition, can be expressed as:

$$\ln\left(\frac{\delta_C T_k^2}{r^2}\right) = \ln\left(\frac{EQA\rho}{R\lambda}\right) - \frac{E}{RT_k} \tag{6A}$$

Eqn. (6A) can be used to determined the activation energy of the coal oxidizing reaction. The previous experimental studies[3-4] using F-K model obtained a critical spontaneous ignition condition for the bituminous coal, namely,

$$\ln\left(\frac{\delta_C T_k^2}{r^{*2}}\right) = 19.56 - \frac{5480}{T_k} \tag{6B}$$

Comparing Eqn. (6A) with Eqn. (6B) and taking the property parameters[3-4] of the coal into Eqn. (6), the pre-exponential factor A is estimated to be 29 $mol\cdot m^{-3}\cdot s^{-1}$, the activation energy E of the coal oxidizing reaction to be 45600 $Jmol^{-1}$.

2.3.3 Natural convection coefficient α Under the research condition, if T_∞ equals to 293K and the following expressions are satisfied,

$$\begin{cases} d = 2r_1 \le 0.50\text{m} \\ 10^4 \le Gr\cdot Pr \le 10^9 \end{cases} \tag{7}$$

where, $Pr=C_{p.a}\mu_a/\lambda_a$ and $Gr=\beta g \Delta T d^3/v_a^2$ are calculated using the value of air at the characteristic temperature of air film. Then, the natural convection coefficient α for the infinite long cylinder can be expressed as,

$$\alpha = 1.34\left(\frac{\Delta T}{d}\right)^{1/4} = 1.127\left(\frac{T_S - 293}{r_1}\right)^{1/4} \tag{8}$$

where, ΔT equals to $T_S - T_\infty$(T_∞=293K); r_1 is the radius of dust layer, in the unit of m.

3. DISCUSSIONS

3.1 The Critical Ignition Conditions for the Dusts around a Power Cable

Taking Eqn. (8), the expression of α, into Eqn. (5) and the relevant parameters of the coal into Eqns. (2) and (5), they are changed into,

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + 7.5 \times 10^7 \exp\left(-\frac{5480}{T}\right) = 0, \quad (r_0 \le r \le r_1), \quad (T_C \le T \le T_S) \tag{9}$$

$$\int_{r_0}^{r_1} 1.74 \times 10^7 r \cdot \exp\left(-\frac{5480}{T}\right) dr + 1.238 = 1.127 r_1^{3/4} (T_s - 293)^{5/4} \tag{10}$$

$$(r_0 \le r \le r_1), \quad (T_C \le T \le T_S)$$

The boundary conditions for the numerical solutions to Eqns. (9) and (10) are,

$$\begin{cases} r = r_0, & T = T_C \\ r = r_1, & T = T_S, \quad -\lambda \frac{dT}{dr}\Big|_{r=r_1} = \alpha (T_S - T_\infty) \end{cases} \tag{11}$$

Under these boundary conditions, Eqn. (9) combined with Eqn. (10) can give the critical spontaneous ignition conditions of the self-heating dusts with an inner heat source, i.e., the correspondent relationship between dust with radius r_1 and dust surface temperature T_S.

The steps for solving Eqns. (9) and (10) are as follows.

Firstly, with the restrictions of Eqn. (11), supposing a given T_S, numerical solutions to Eqn. (9) can give a series of temperature distributions for different r_1, namely, the relations of T(r) against r. For instance, Figure 3 shows the T(r)-r relations when r_1=0.2m and r_1=0.1m.

Secondly, iteratively integrating Eqn.(10) and using Runge-Kutta's difference differential method can obtain a certain T_S and r_1 that satisfy both Eqn.(9) and Eqn.(10).

To get calculating results quickly, variation of parameters can be used roughly to estimate the ranges of T_S and r_1.

Figure 4 shows the relations between the temperature of dust surface and the different dust radii, which are obtained from the numerical solutions of Eqns. (9) and (10).

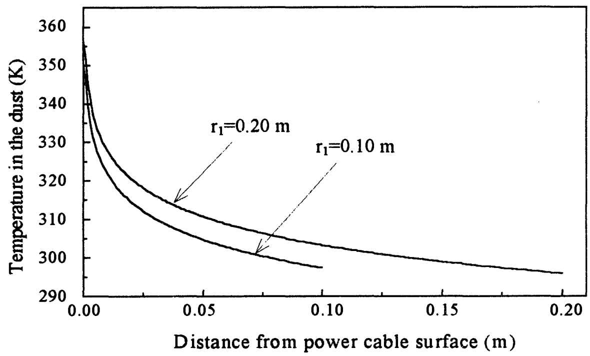

FIGURE 3 Temperature distributions in the dust-layers for r_1=0.20m (upper line) and r_1=0.10m(lower line); power cable radius $r_0=0.69\times10^{-3}$m.

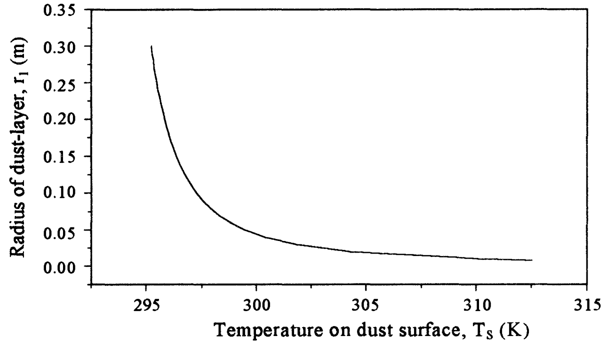

FIGURE 4 The relations between the temperature of dusts surface and the different radii of the dust (power cable radius $r_0=0.69\times10^{-3}$m).

This paper has obtained approximate critical spontaneous ignition conditions for the coal dusts around a power cable, saying, different critical spontaneous ignition temperature(T_S) on the dust surface for correspondingly given dust radius r_1, as shown in Table 1.

TABLE 1 Critical spontaneous ignition temperature on dust surface against its critical spontaneous ignition radius of dusts (power cable radius $r_0=0.69\times10^{-3}$m).

T_S (K)	296	298	303	313
$r_1\times10^3$ (m)	182	77.5	24.4	7.7

Apparently, if the ambient temperature is set to be different values (not to be 293K as supposed in the previous calculations), critical spontaneous ignition conditions will be changed, saying, the correlation between the dust surface temperature and its correspondent critical spontaneous ignition radius of dust will be different for a given power cable radius.

3.2 The Critical Ignition Conditions for the Dusts without Inner Heat Source

The critical spontaneous ignition conditions for the dusts without any inner heat sources can be calculated through F - K model, by taking δ_C=2.00 (for a cylinder sample) into Eqn.(6). So, critical temperature(T_k) of spontaneous ignition corresponds to a critical spontaneous ignition radius r_1' of the dust cylinder. Calculated results are listed in TABLE 2.

TABLE 2 The critical spontaneous ignition conditions for the dusts without any inner heat sources calculated by Frank - Kamenetskii model

T_k (K)	296	298	303	313
$r_1' \times 10^3$(m)	248	235	205	159

3.3 Comparison

Under the same ambient temperature, the critical spontaneous ignition radius (r_1)of the dusts with an inner heat source (around the power cable) is shorter than that (r_1') of the dusts without any inner heat sources. These results show that the dusts on the surface of a working power cable are very easy to ignite spontaneously.

The dusts with an inner heat source will ignite spontaneously when T_S is given and r is greater than r_1. This will occur if the dust layer around the cable is getting greater due to the cable works in dusty surroundings. In these circumstances, the heat conduction resistance of the dust layer increases, the inner temperature increase gives rise to a quicker oxidizing reaction of the dust. Finally, the dust may undergo a thermal runaway that probably causes a power cable fire.

When Temperature on the external surface of dusts around the working power cable is greater than calculated critical ignition temperature (T_S) for the given radius r_1, the dust will ignite spontaneously. This will occur if the dusts in a power cable trench block the heat dissipating holes, which worsens the convection heat exchange between the dusts and the ambient air.

A small power cable acted as a weak inner heat source of dusts around the cable has a great influence on the self-heating behaviour of the dusts, refering to TABLE 1 and 2.

4. CONCLUSIONS

1) For the same geometric shape of dusts, the critical spontaneous ignition radius of the dusts on the working power cable surface is shorter than that of the dusts without any inner heat sources. In comparison with the dusts without any inner heat sources, the dusts on the

working power cable surface will ignite spontaneously at a lower temperature, referring tables presented earlier.

2)The working power cable acted as an inner heat source of the dusts around the cable plays an important role in the self-heating of the dusts. Inversely, the spontaneous ignition of the dusts around the cable may cause a power cable fire.

3)This work has not been evaluated experimentally, the findings may not match the practical situation where the dust shape is sharply different from a cylinder. This preliminary study is to help with understanding how a weak inner heat source influences on the self-heating behaviour of the dust around the cable.

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