

# Fire Spread Model in a Porous Medium

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## ABSTRACT

A new algorithm for fire spread simulation is proposed by using a dynamic tool of the flame front change and a local spread model. The progression of fire, represented by a closed contour, is based on the determination of a parametric function. In order to compute this new parametric function, it is necessary to calculate the scalar field of the rate of spread at some points of the fire contour. A physical model has been introduced and is mainly based on the Cekirge model. This takes into account external conditions i.e. the wind, the topographic conditions and the fuel characteristics. This model provides then results with a moving boundary problem and can be used to evaluate forest fire propagation in real time. Validations are presented in order to demonstrate the soundness of this new approach.

**KEYWORDS:** forest fire, porous medium, unsteady propagation, dynamic tool.

## NOTATION

$a = \frac{\sigma_f \beta}{4}$	absorption coefficient ( $m^{-1}$ )
$a_k$	complex Fourier series coefficients
$C_s^0$	specific heat of dry fuel (J/kg/K)
$C_w$	specific heat of water (J/kg/K)
$D_f$	reaction zone depth (m)
$F_v$	view factor
$H$	flame height (m)
$h_c$	convection coefficient ( $W/m^2/K$ )
$I_R$	reaction intensity ( $W/m^2$ )
$L_f$	flame length (m)

$L_w$	latent heat of vaporization of water (J/kg)
$M_f$	moisture content, % of oven-dry fuel
$N$	harmonic number
$R(t)$	position of fire front with time (m)
$t$	time (s)
$T$	temperature (K)
$T_0$	ambient temperature (K)
$V$	rate of spread (m/s)
$U$ or $U_w$	wind velocity (m/s)
$Z$	complex variable

### Greek letters

$\alpha$	wind coefficient
$\beta$	porosity of the fuel
$\delta$	height of fuel bed (m)
$\epsilon$	emissivity
$\theta$	argument of the complex number $Z$ (rad)
$\rho$	density ( $\text{kg/m}^3$ )
$\sigma$	Stefan-Boltzmann constant ( $\text{W/m}^2\text{K}^4$ )
$\sigma_f$	fuel particle surface-area-to-volume ratio ( $\text{m}^{-1}$ )
$\tau_R$	reaction time (s)
$\varphi_f$	flame angle, measured from vertical (rad)
$\varphi_s$	slope land angle (rad)

### Subscripts

$e$	embers
$f$	flame
$fe$	flame-embers
$g$	gas
$ig$	ignition
$s$	solid

## INTRODUCTION

The understanding and the control of the development of wildland fire constitute a true challenge for those in charge of protection and security, but also for the whole scientific community. These fires cause great damage to the environment, including wildland itself, but also have social and economic consequences. Research into protection and safety is generally structured at several levels. First of all improved management of forests by the installation of fire breaks. Moreover, methods and tools to evaluate the fire risk, fire fighting policies, air and terrestrial fire fighting management as well as the tactics of fire fighting, all contribute to the efficiency of the fire services and the improvement of the protection of forests [1]. Thus, for 25 years, the number of small and medium fires has decreased continuously in France. However, the recent fires in Indonesia, in the United-States and even in France show, if it is necessary, that when there are difficult weather conditions, fire can progress dramatically. From the security point of view, research is turning towards modelling software to deal with

not only the physical phenomena but also the characteristics of the medium (topographic, climatic...) in which fire develops. Despite the increase in computer power, it is still illusory to model the development of a forest fire in real time by taking into account the whole physical process. A forest fire is generally a dynamic phenomenon that changes its properties and behaviour in time and fire behaviour is determined by complex heat transfer and heterogeneous combustion processes : the rate of fire spread depends on the fuel itself (characteristics, moisture and porosity), on the wind interaction and on the slope. Recently, for instance, Mc Donough & al. [2] presented a new modelling approach for simulating wildland fires and provided qualitative comparisons of flow fields and thermal plume structures, both instantaneous and time averaged, for different cross wind and porous medium compactness with an LES turbulence modelling. This opens the way to a wide spectrum of potential applications including fire spread predictions in large scale urban fires as well as in various types of wildland fire scenarios. Nevertheless, this type of modelling cannot estimate in real time the total extent of the fire propagation. For this reason, a multitude of empirical, analytical or numerical models in 1D or 2D continue to be interesting. Weber [3] made an exhaustive review of the essential ingredients needed to make a mathematical modelling of fire spread through a fuel bed.

Statistical models are a description of test fires and make no attempt to involve physical mechanisms. Empirical models are the first step in the understanding of the spread but are not entirely satisfactory from a scientific and a practical point of view [3]. These models are based on the energy balance between the flame and the fuel upstream which is not yet burning. The use of semi-empirical models such as the model of Rothermel [4] and its derivatives (Frandsen and Andrews [5], Green & al [6], Albini [7]) consist of the application of physical parameters (fuel particle size, moisture, etc.) which are combined together using physical analysis to produce components of a formula for the rate of spread. The Rothermel model takes into account the characteristics of fuel, the water content, the wind, the slope or the solar phenomena.

Weber's model takes into account the different modes of heat transfer in the fuel bed and attempts to predict the rate of fire spread using more fundamental physical and mathematical means. Predicting the rate of fire spread is mainly characterised by a heat transfer differential equation based on an energy balance. This equation contains two unknowns : the temperature profile and the flame front position, the latter being fundamentally linked to the rate of spread ( $V$ ) of the fire. With the assumption that temperature profile behaves like a "travelling" wave at velocity  $V$ , Weber obtained an analytical solution for the rate of spread. Furthermore, problem of the fire front propagation can be considered as a moving boundary problem while accounting for unsteady rate of spread. Cekirge [8] proposed such a model providing changes in time and the temperature field in front of the fire. This would provide information when considering the use of fire extinction methods (trenching, water or chemical spraying, ...).

With the rise of graphic data processing, the visualisation of the results of fire behaviour simulation became a subject for research in its own right. These models indeed make it possible to anticipate the flame front propagation and thus to quantify and to locate the backup facilities as efficiently as possible. There is a multitude of graphic models based on various theories. The elliptic models are based on fire propagation following the shape of an ellipse. This contour is then compared to a succession of independent ignition points which become origin points of new ellipses whose envelope forms a new fire contour. This is the well-known Huygens principle applied to the propagation of forest fire. The propagation velocity at each

point is given, in general, by empirical or semi-empirical models. A software was developed recently in the United States by Finney [9] to develop the control of directed fires and the management of fires over long periods (> 24 hours). This software integrates a graphic evaluation of the propagation using the Huygens principle, a Geographic Information System (GIS) and a quantitative evaluation of the propagation based on the Rothermel model. The use of the cellular automata is a new way of modelling of forest fires [10]. Cellular automata models assume a grid pattern with mosaics of cells. Traditionally, these mosaics are based on cells of identical size and shape. By using this particular model of propagation, geographic contamination by fire may be estimated. The principal disadvantages of this modelling seem to be the calculation time and the deformation of contour by the geometry of the cells. The bibliographical review we have just discussed has made it possible to highlight different modelling and to underline the difficulties still to be overcome today. An approach combining, at the same time, graphic development and a better physical model seems to be the way forward for future research.

The aim of this paper is to present a modelling research including these two considerations. As Plourde already stated [11], PROLIF mathematical algorithm has been developed to model the propagation of a moving flame front in forests or grassland. This includes a rate of spread estimation. This model separates the propagation of the flame front from the rate of spread calculation itself. The originality of this step is to characterise the closed contour by a complex trigonometrically parametric function. A module makes it possible to calculate a contour with time  $t$  and to deduce the function at the time  $t + \Delta t$ . A second module makes it possible to evaluate a rate of spread vector all along the closed contour. This physical model may be adapted in order to take into account for example the specific medium, the local environment, or the climatic conditions. Previous work presented the PROLIF tool with a constant rate of spread obtained from Weber's model.

## **LOCAL PROPAGATION MODEL**

As mentioned above, different models mean that the flame front propagation can be simulated. Mathematical models are based on empirical methods, on physical or on a combination of these two methods. The modelling presented in this paper belongs to the third category. Indeed, the local characteristics of the flame front are calculated considering physical analysis as well as semi-empirical laws. However, in order to provide a real time simulation, some parameters of the combustion zone are assumed to be known such as the flame temperature above and inside the porous bed. Then, the problem consists at primarily solving the energy conservation equation in the fuel bed in the presence of wind and slope.

### **Physical model**

With local scale, the flame front is characterised by the rate of spread  $V$ , by the reaction zone and its geometry (reaction intensity  $I_R$ , length of flame  $L_f$ , flame front depth  $D_f$ , flame tilt angle  $\varphi_f$  and flame-embers tilt angle  $\varphi_{fe}$ , etc...). Thus, in order to determine the front propagation, it is sufficient to carry out an energy balance of various heat transfers considered (fig.1a) i.e. radiation and convection (transfers by conduction are not taken into account).

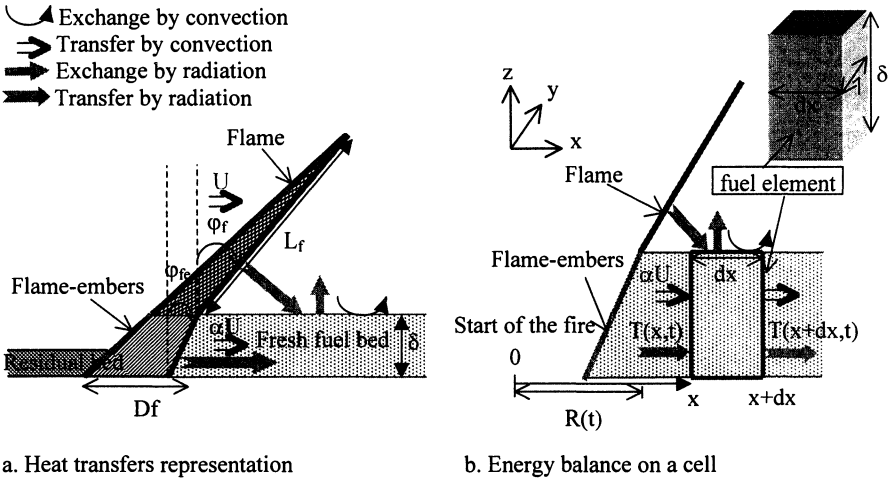


FIGURE 1. Present heat transfers at the time of a bed fire

Heat transfer by flame radiation occurs in a transparent medium (air) whereas flame-embers radiation is propagated through a semitransparent medium, assimilated to a porous medium. Transfer by convection is taken into account with the assumption of a flow inside the porous medium which can be characterised by a wind velocity  $U$  and a wind coefficient  $\alpha$  ( $0 < \alpha < 1$ ). Moreover, various assumptions are taken as read in this physical model. Firstly, the fuel bed is an uniform porous medium of compactness  $\beta$  constituted of elementary particles. An elementary particle compared with a black-body is supposed to ignite itself when it has absorbed sufficient energy corresponding to ignition temperature  $T_{ig}$ . Flame temperature and embers temperature are supposed known and constant, and finally, fuel moisture content is taken into account in the expression of calorific capacity of the fuel (Weber [12]) :

$$C_s = C_s^0 + M_f (C_w (373 - T(x, t = 0)) + L_w) / (T_{ig} - T(x, t = 0)) \quad (1)$$

Energy balance carried out on a cell of fresh fuel (fig.1b) is inspired by the one-dimensional model presented by Cekirge [8]. The conservation of energy for this fuel element is expressed in the following way:

$$(\beta \rho_s C_s + (1 - \beta) \rho_g C_g) \frac{\partial T}{\partial t} + (1 - \beta) \rho_g C_g \alpha U \frac{\partial T}{\partial x} = G(x, R(t), T) \quad (2)$$

where:

$$G(x, R(t), T) = \alpha_1 (x - R(t)) T^4 + \alpha_2 T + \alpha_3 (x - R(t)) + \alpha_4 (x - R(t)) T_r^4 + \alpha_5 \quad (3)$$

with:

$$\alpha_1 = -\frac{\sigma \epsilon_f}{\delta} F_v (x - R(t)) - \frac{\sigma}{\delta} \quad (4)$$

$$\alpha_2 = -\frac{h_c}{\delta} \quad (5)$$

$$\alpha_3 = \sigma \epsilon_{fc} a \Gamma_c^4 e^{-a(x-R(t))} \quad (6)$$

$$\alpha_4 = \frac{\sigma \epsilon_f}{\delta} F_v (x - R(t)) \quad (7)$$

$$\alpha_5 = \frac{\sigma T_0^4 + h_c T_0}{\delta} \quad (8)$$

The initial condition of the problem at  $t = 0$  and the boundary condition at  $x=R(t)$  are expressed respectively  $T(x, 0)=T_0$  and  $T(R(t), t)=T_{ig}$ . Thus, this balance makes it possible to obtain an unsteady rate of spread due to the moving boundary condition in the course of time as well as the temperature distribution according to direction  $x$  at the time  $t$ . Moreover, the energy conservation equation formulation obliges us to determine a certain number of parameters such as the absorption coefficient  $a$  and the view factor  $F_v$ . The definition of the absorption coefficient  $a$  of the radiation in a semitransparent medium has been defined as the reverse of the average course of the radiation inside the semitransparent medium (Committee on Fire Research [13]). The view factor  $F_v$  has been taken as the Steward formulation [14] :

$$F_v = 0.5 \left( 1 + \frac{L_f \sin(\varphi_f) - (x - X(t))}{\sqrt{L_f^2 + 2(x - X(t))L_f \sin(\varphi_f) + (x - X(t))^2}} \right) \quad (9)$$

The combustion zone is characterised by the energy which it releases called reaction intensity  $I_R$  and by its geometry (length of flame  $L_f$ , flame front depth  $D_f$ , etc...). The reaction intensity  $I_R$  is given using the empirical laws of Rothermel [4] obtained at the time of various experiments (poplar shavings, pine needles, etc...) as :

$$I_R = -\frac{dw}{dt} h \quad (10)$$

with:  $\frac{dw}{dt}$  (in  $\text{kg/m}^2/\text{s}$ ) is the mass loss rate per unit area in the flame front and  $h$  (in  $\text{J/kg}$ ) is the combustion heat. The reaction zone depth  $D_f$  is given using the reaction time  $\tau_R$  corresponding to the time necessary for the total combustion of an elementary particle. It is defined starting from an empirical law of Rothermel:

$$\tau_R = \frac{\eta_D}{\Gamma} \quad (11)$$

with  $\eta_D$  and  $\Gamma$  (in  $s^{-1}$ ) corresponding to the combustion zone efficiency and the reaction velocity respectively. The combustion zone efficiency is taken equal to 1 and the reaction velocity is calculated by empirical laws (Rothermel [4]) which will not be exposed in this paper. Then, the flame front depth is expressed by:

$$D_f = V\tau_R \tag{12}$$

For the emissivity of the flame  $\epsilon_f$ , Thomas [15] proposed a formula involving the front depth and an attenuation coefficient:

$$\epsilon_f = 1 - \exp(-K D_f) \tag{13}$$

An experiment carried out by Woollicroft [16] gave an attenuation coefficient  $K=0.16$ . In order to define the emissivity of the flame-embers located inside the fuel bed  $\epsilon_{fe}$ , it is only necessary to add the embers radiation and we obtain :

$$\epsilon_{fe} = 1 - \exp(-0.16(D_f + 0.375)) \tag{14}$$

The length of flame  $L_f$  used in this model was defined by Thomas [17] using a dimensional analysis:

$$L_f = 2.6641 \cdot 10^{-3} (I_R D_f)^{\frac{2}{3}} \tag{15}$$

Moreover, flame or flame-embers tilt angle is a function of the ratio of inertias to gravity forces. We can thus clarify this by using the Froude number (Albini [18]) such as:

$$(\tan \varphi)^2 = 1.5 Fr \quad \text{where } Fr = \frac{U^2}{gH} \tag{16}$$

In order to obtain a horizontal configuration, it is sufficient to transform the slope  $\varphi_s$  using the Albini equation into a "fictitious wind"  $U_s$  which is quite simply added to wind  $U_w$  (fig.2).

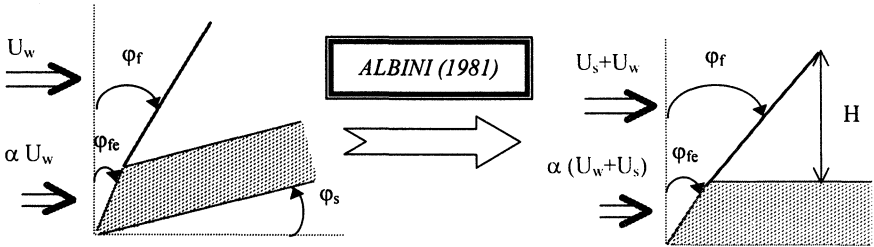


FIGURE 2. Transformation of the slope angle  $\varphi_s$  into a "fictitious wind"  $U_s$

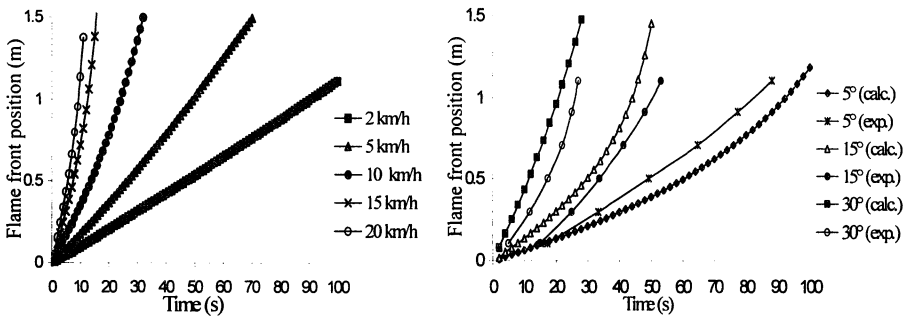
Thus, by dichotomy, we obtain two new values of the angles  $\varphi_f$  and  $\varphi_{re}$  taking into account the slope and the wind.

All these previously stated parameters are dependent on each other. They must then be computed until convergence is reached for all these parameters.

### The influence of wind and slope conditions on propagation

In this part, we are going to look at the influence of external conditions, i.e. wind and slope effect, on the rate of spread. Under certain conditions, wind and topography strongly activate forest fires. In fact, they emphasise heat transfer by convection inside the vegetable bed. Thus, to know how these two parameters act on the flame propagation, the change of fire front position with time was studied for various values of wind and slope (fig. 3a and 3b).

Firstly, it can be observed that the position of the flame front behaves initially as a polynomial function of time then becomes a linear function of time. However, for the low values of wind, the position of front is only linear with time without polynomial behaviour. Indeed, this means that the propagation unsteadiness is rather reduced. Then, rate of spread is about constant. Moreover, figure 3a shows that up to 10 km/h, the wind increases strongly propagation velocity. Above 10 km/h, the influence of this parameter on the front speed decreases. The same observation could be made for the slope effect since wind and slope act in the same way on propagation. Thus, the assumption can be made that wind and slope allow an acceleration of flame front in a certain field of values and that apart from this field, these two parameters do not have any more influence or could even have a beneficial influence to extinguish fire. Experiments carried out by Naville [19] were related to figure 3b but they only allowed more or less to validate our local model in the presence of topography. Thus, the assumption made previously should be checked by complementary experiments.



a. Influence of wind on propagation

b. Influence of slope on propagation (experiment and calculation)

FIGURE 3. Influence of external conditions (wind and slope)



## FIRE CONTOUR TOOL

The progression of the fire line, represented by a closed contour, is based on the determination of a parametric function. Scalar field  $V(X, Y, t)$  is known for each point of the plan and at each instant  $t$  using the local model described above. As the progression occurs outwards from the closed contour, the space covered is consequently unburned, except in the particular case where the line of fire is closed.

A mathematical representation of the closed contour can be extrapolated with a Parametric Trigonometric Representation of Fourier (PTRF) which is expressed as:

$$Z(\theta, t) = \sum_{k=-N/2}^{N/2} a_k(t) e^{ik\theta} \quad (17)$$

in which the coefficients  $a_k(t)$  correspond to the complex coefficients of the Fourier series. The use of this PTRF has the main advantage of producing a good representation of the shape of the initial contours. If the shapes to be represented were angular, the corresponding Fourier series need only to be raised to a higher order.

The closed contour, through the PTRF, is known at instant  $t^n$  and the problem consists of determining the contour at instant  $t^{n+1}$ , i.e. that the problem now is to determine all the  $(a_k^{n+1})$  coefficients. The proposed numerical procedure is linked to working in the domain phases i.e. that the  $(a_k)$ 's are looked for to obtain a new contour at  $t^{n+1}$  and not the contrary, which would consist in looking for new discrete locations whose  $(a_k^{n+1})$ 's would be deduced. The mathematical system is as follows:

$$\frac{\partial G}{\partial t} = A \cdot G \quad (18)$$

with:  $G$ , vector column  $G^T = (a_{-q}, \dots, a_0, \dots, a_q)$ ,  $N = 2q$  and  $A$ , square matrix  $N+1, N+1$  such that  $a_{j,k} = (k-q-1) \alpha_{j-k}$ .

It is to be noted that the choice of time step does not depend as much on criterion stability but on criterion precision. Indeed, the progression being unlimited, the  $(a_k)$  coefficients are not bounded. The choice of the time step would be dictated more by the degree of inhomogeneity of the bidimensional framework on which the closed contour propagates.

After a new perimeter has been formed, it is necessary to identify and remove anomalies in the new perimeter. There are two types of anomaly, a rotation nearly always occurs at a moderately concave point due to the consistent order in which new points are evolved ; overlaps may also have occurred where regions of the front have crossed over after surrounding an area of low fire spread or a fire break. Both cases must be identified and removed.

## VALIDATION OF CODE PROLIF WITH A TEST FIRE

As Plourde [11] already made, the simulation results were compared to the results of an experiment subjected to external conditions called "controlled fire". Thus, we wanted to know if code PROLIF gave a good approximation of fire contours just by knowing the characteristics of plant, the topography of ground and the weather conditions estimated during the test fire. Then, rates of spread have been evaluated at discrete points and PROLIF provides fire contours change.

Test fires are spring fires ignited and controlled by firemen. Thus, they make it possible to understand better the phenomenon of fire propagation without incurring the slightest danger. At the time of the controlled fire considered, the surface of 1000 m<sup>2</sup> to be burned was squared thermocouples and had an average slope of 20°. The vegetation of this area consisted of two vegetable layers of which the first was made up of grass of a height from 5 to 10 cm and the second of broom of a height from 50 to 65 cm. During the fire, wind speed was about 27 km/h. The fire was lit at the bottom of the slope in the form of a fire line of 20 m length and was propagated in the direction of the slope, also corresponding to the direction of the wind. The area burned in 65 seconds and average speed of burning was estimated at 0.76 m/s (fig. 4). At the time of the controlled fire, the flame front unceasingly underwent speed fluctuations. This phenomenon can be due to the change of wind intensity or the more significant presence of grass at the bottom of slope. Fire contours resulting from simulation were drawn every 8 seconds on figure 4.

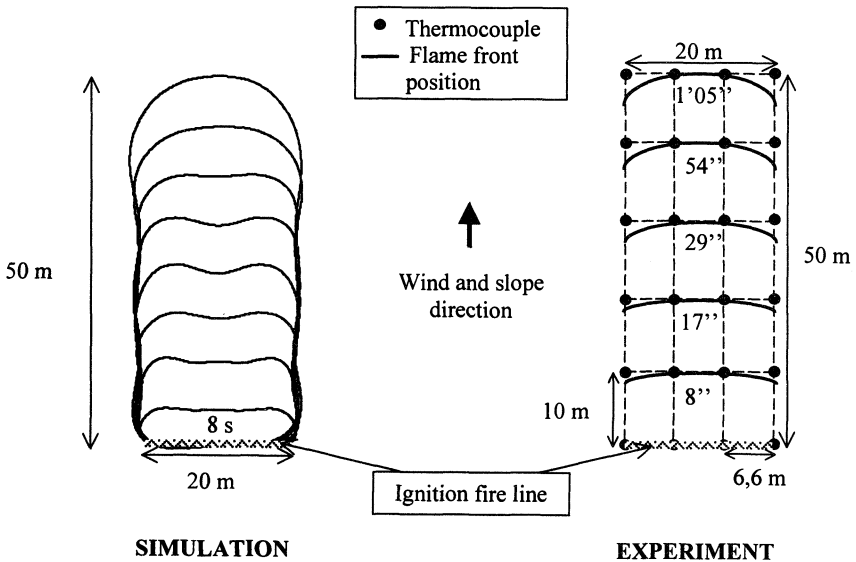


FIGURE 4. Comparison between simulation and experiment for the controlled fire

The rate of spread became very quickly steady. Indeed, starting from  $t = 16$  s, speed was already constant and equal to 0.78 m/s. If experiment and modelling are compared, it can be observed that the simulated flame front did not undergo such a strong acceleration as the real flame front at the start of the fire. This is due to the explanations given previously. On the other hand, at the end of the directed burning, the two flame fronts (simulated and real) coincide. Then, experiment results are always subjected to the risks of meteorology and the heterogeneity of vegetable cover. To simulate real fires, it would thus be necessary that code PROLIF be able to modulate the input data during calculation by making a grid of the surface to be burned. Moreover, there is always an overlapping of contours when at certain points of these contours, the rate of spread is null. Then, it produces some disturbance in the contour but the general shape can be evaluated.

## CONCLUSION

Development of such a tool such as PROLIF is necessary in order to improve the understanding of wildland fire propagation and as firemen need real-time data, new software must be able to evaluate rate of spread and fire contours. In fact, although computers are constantly progressing, assumptions are still needed in order to carry out real-time analysis. Fire is defined as a closed contour propagating outwards at a velocity directly linked to the physical process of combustion. The main idea is then to separate the contour evaluation and the energy equation itself. A local model has been developed and is able to characterise change of rate of spread with time whereas contour evolution is modelled by a complex Fourier series function. In the local model, wind and slope effects have been studied. In comparison with the calculation, the rate of spread obtained with the local model are in agreement with the experiment. Furthermore, contours with PROLIF are also in concordance with fire test contours. In the future, it would be necessary to take into account weather, topographic data and vegetable cover characteristics change, during calculations. PROLIF is a code which can be easily coupled with other codes allowing to modulate the input data. Thus, a cartography of the risk zones could be integrated in order to take account of topography and vegetable cover variations. Moreover, a code developed by the Mathematical Center of Applied Modelling of the Technology Institute of Victoria (Australia) and called "NUATMOS" could be also coupled with PROLIF in order to calculate three-dimensional field wind speed with a complex topography.

## REFERENCES

1. Sauvagnargues, S., "Contribution à la mise en œuvre d'un Système d'Information Géographique appliquée à la sécurité civile pour la prévention et la lutte contre les feux de forêts", PhD Thesis, University of Aix-Marseille I, 1998.
2. Mc Donough, to be published.
3. Weber, R. O., "Modelling fire spread through fuel beds", Prog. Energy Combust. Sci., Vol 17, pp 67-82, 1991.
4. Rothermel, R.C., "A mathematical model for predicting fire spread in wildland fuels". Forest Service USDA Res. Pap. INT - 112, 1972.

5. Frandsen, W.H. & Andrews, P.L., "Fire behaviour in nonuniform fuels". USDS Forest Service Paper INT – 232,1979.
6. Green, D.G., House, A.P.N. & House, S.M., "Simulating spatial patterns in forest ecosystems", Math. Comput. Simul., 27: 191-198, 1985.
7. Albini, F.A., "A model for fire spread in wildland fuels by radiation", Combust. Sci. Technol., 42 :229-258, 1985.
8. Cekirge, H.M., "Propagation of fire fronts in forests", Comp. & Maths. With Appls, Vol.4, 325-332, 1978.
9. Finney, M.A., "Farsite v2.0, users guide and technical documentation", Systems for Environmental Management, p.116, 1996.
10. Gonçalves, P.P., Diogo, P.M., "Forest fire propagation modeling: a new methodology using cellular automata and geographic information systems", Coimbra Proc. 2<sup>nd</sup> Int. Conf. Forest Fire Research, 1: 209-219, 1994.
11. Plourde, F., Doan-Kim, S., Dumas, J.C., Malet, J.C., "A new model of wildland fire simulation", Fire Safety Journal, 29: 283-299, 1997.
12. Weber, R.O., "Analytical models for fire spread due to radiation", Combustion and Flame, 78, pp. 398-408, 1989.
13. Committee on Fire Research, U.S. Nat. Acad. Sci., Nat. Res. Council. Pub.949, 1961.
14. Steward, F.R., "Radiative heat transfer associated with fire problem", heat transfer in fires: Thermophysics Social Aspects, Economic Impact, (Edited by P.L. Blackshear) John Wiley, New York, 1974.
15. Thomas, P.H., "The contribution of flame radiation to fire spread in forests", Fire Res. Sta., Fire Res. Note 594, 19, 1965.
16. Woollicroft, M.J., "Notes on forest fire fieldwork", Joint Fire Research Organization Fire Research, Notes N°740, 1969.
17. Thomas, P.H., "Some Aspects of the growth and spread of fire in the open", Forestry, 1967.
18. Albini, F.A., "A model for the wind-blown flame from a line fire", Combustion And Flame, 43: 155-174, 1981.
19. Naville, L., "Etude expérimentale et modélisation de la propagation d'une flamme sur un combustible poreux: applications aux feux de végétation", PhD Thesis, University of Aix-Marseille II, 1997.