Simple Method to Predict Fire Resistance of Composite Columns

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ABSTRACT

A simplified numerical method to predict the fire resistance of concrete filled tubular composite columns is presented. The temperature analysis of the composite column is formulated as a one dimensional Galerkin FE-solution of the heat conduction equation. The simple design method of the Eurocode 4 has been applied in the calculation of the load-bearing capacity of the tubular composite columns. Calculated temperatures and load-bearing capacities are compared to the test results of concentrically loaded circular and square tubular columns filled with reinforced and non-reinforced concrete.

KEYWORDS: fire resistance, composite columns, heat conduction, Eurocodes

INTRODUCTION

Numerous research projects and standard fire tests have been conducted on different types of composite columns in Germany, France, Belgium, England and Canada. Various kinds of design methods for composite columns have been developed in the literature [1-2,4-11]. In the Eurocode 4, Design of Composite Steel and Concrete Structures, Part 1.2 [4] calculation methods for the structural fire design are divided into three categories, tabulated data design methods, simple and advanced calculation methods.

An Excel-based FE-code has been written by the authors for the thermal and structural analysis of composite columns. The thermal analysis part of this code was developed in the Nordtest project 1381-98 to calculate heat transfer in an axisymmetric duct [14]. The program was further developed for the purposes of the seminar held at Helsinki University of Technology on the design of steel/concrete composite structures. Structural analysis based on the simple design method of the Eurocode 4 was adapted in the computer program for non-reinforced circular columns [12]. Here the temperature and structural analysis is extended also for reinforced square tubes.

The two dimensional temperature field is solved using a one dimensional Galerkin FEM formulation of the heat conduction equation. The dimension reduction of the 2-D heat transfer problem to a 1-D problem is achieved assuming that the isothermal contours of the temperature field are either circular or rectangular. In the axisymmetric case of circular columns a circular contour assumption is exact if the temperature field around the column is uniform. In the case of square columns a rectangular contour assumption is often a suitable approximation for engineering purposes.

CALCULATION OF TEMPERATURE

Solution of General Heat Conduction Problem

The general two-dimensional partial differential equation of heat conduction is to be solved in order to get the accurate transient temperature field $T(x,t)$ in a given material region. The field equation

$$\rho\ c\ \dot{T}(x,t) = \vec{\nabla}\cdot\left(\lambda\vec{\nabla}T(\vec{x},t)\right) + r(\vec{x},T) \tag{1}$$

is the heat conduction equation with $r(x,T)$ as an arbitrary source term. The Fourier heat conduction constitutive equation is assumed. Equation (1) is complemented with the appropriate initial-boundary conditions to get a well-posed problem. Using the standard FE-approach [3] one solves the variational form of the problem (1)

$$\int_\Omega \rho\ c\ \dot{T}\ v\ d\Omega + \int_\Omega \lambda\ \vec{\nabla}T\cdot\vec{\nabla}v\ d\Omega = \int_\Omega r\,v\ d\Omega - \int_{\partial\Omega_q} \vec{q}\cdot\vec{n}\ v\ d\Gamma \tag{2}$$

by choosing the temperature field approximation $T^e(x,t) = \mathbf{N}^e(x)\mathbf{T}^e(t)$ and the test function $v_i(x) = N_i^e(x)$, where the basis functions N_i^e are linear.

The semidiscretization of the heat conduction equation (2) produces the non-linear initial value problem

$$\mathbf{C}(t,\mathbf{T})\dot{\mathbf{T}}(t) = \mathbf{f}(t,\mathbf{T}) - \mathbf{K}(t,\mathbf{T})\mathbf{T}(t),\ t > 0 \tag{3}$$

$$\mathbf{T}(0) = \overline{\mathbf{T}}_0,\quad t = 0,$$

where $\mathbf{T}(t)$ is the global vector of the unknown temperatures of size $n\ x\ 1$.

Equation (3) is a set of $n\ x\ 1$- non-linear ordinary differential equations. Notice that the right hand side in the equation (2) corresponds to the nodal flux vector $\mathbf{f}(t,\mathbf{T})$, which contains the boundary terms and the source terms. Natural boundary conditions are already included in the variational form (2). The essential boundary conditions are taken into account during the solution process of the initial value problem. The global matrices and vectors are assembled using standard FE-assembling techniques. In the axisymmetric case (see Fig. 1a) ($f(x,y,z;t) = f(r,z;t)$), when linear 2-noded elements are used, the following element conductance matrix is obtained

$$K_{ij}^e = \int_{\Omega^e} \lambda(T(\mathbf{x}))\ \vec{\nabla}\mathrm{N}_i(\mathbf{x})\cdot\vec{\nabla}\mathrm{N}_j(\mathbf{x})\ d\Omega\ =\ 2\pi\ \Delta l \int_{r_1^e}^{r_2^e} \lambda(T(r))\ N_{i,r}(r)\ N_{j,r}(r)\ r\,dr \qquad (4)$$

and also the element capacitance matrix

$$C_{ij}^e = \int_{\Omega^e} \rho(T(\mathbf{x}))\ c(T(\mathbf{x}))\ \mathrm{N}_i(\mathbf{x})\ \mathrm{N}_j(\mathbf{x})\ d\Omega = 2\pi\ \Delta l \int_{r_1^e}^{r_2^e} \rho(T(r))\ c(T(r)) N_i(r)\ N_j(r)\ r\,dr \ (5)$$

and element nodal heat flux vector

$$f_i^e = -\int_{\partial\Omega_q} \vec{q}\cdot\vec{n}\ \mathrm{N}_i(\mathbf{x})\ d\Gamma = -2\pi\ \Delta l \begin{pmatrix} r_1^e\ q_{n,1} \\ r_2^e\ q_{n,2} \end{pmatrix}\Bigg|_{\Gamma_q^e \cap\, \partial\Omega_q} \qquad (6)$$

The standard basis functions of a 2-noded linear element $N_1(r) = 1-(r-r_1^e)/a^e$ and $N_2(r) = (r-r_1^e)/a^e$ are used. The element volume is $d\Omega = 2\pi\ \Delta l\ rdr$, with Δl as the thickness of the element slice in the z-axis-direction. The radial length of the element, number e, is $a^e = r_2^e - r_1^e$.

In the case of a square and rectangular tubular column an accurate temperature analysis is two dimensional. Here an approximate one dimensional analysis is presented. It is assumed that the isothermal contours of the temperature field are square (Figure 1b). Using linear elements with 2 nodes the following element conductance and capacitance matrices are obtained:

$$K^e{}_{ij} = \Delta l \int_{x_1^e}^{x_2^e} \lambda(T(x))\ N_{i,x}(x)\ N_{j,x}(x)\ A(x)\ dx \qquad i,j = 1,\ 2 \qquad (7)$$

$$C^e{}_{ij} = \Delta l \int_{x_1^e}^{x_2^e} \rho(T(x))\ c(T(x)) N_i(x)\ N_j(x)\ A(x)\ dx \quad i,j = 1,\ 2 \qquad (8)$$

and $\mathbf{f}^e{}_i = -\ \Delta l \begin{pmatrix} A(x_1^e)\ q_{n,1} \\ A(x_2^e)\ q_{n,2} \end{pmatrix}\Bigg|_{\Gamma_q^e \cap\, \partial\Omega_q}$, respectively. The standard basis functions of a linear element are used. The element volume is $d\Omega = \ \Delta l\ A(x)\ dx$. The length of the element e in the radial direction is $a^e = x_2^e - x_1^e$. Here $A(x) = 8\ x$ is the length of perimeter of the square contour at x.

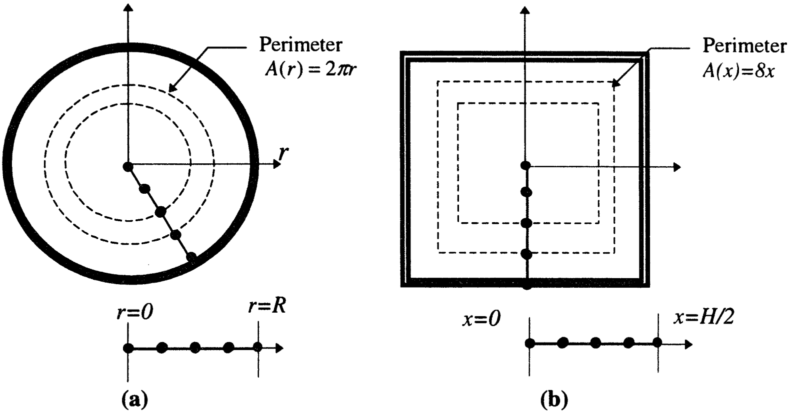


FIGURE 1. a) Circular concrete-filled steel tubular column and the axisymmetric discretization b) square tubular column and an approximate one-dimensional discretization.

Boundary Conditions

In the special case of a cylindrical composite column the following convection and radiation boundary condition (Neumann) is used at the boundaries of the solution domain;

$$\begin{aligned} &q_{n,1}(r=0)=0 \\ &q_{n,2}(r=R)=-h_2(\bar{T})\big(T_f(R)-T_s(R)\big)-\varepsilon_2\sigma\big(T_f^{\,4}(R)-T_s^{\,4}(R)\big) \end{aligned} \tag{9}$$

where T_f is the temperature of the furnace hot gases around the column. Similar boundary conditions are used also in the approximate square solution.

$$\begin{aligned} &q_{n,1}(x=0)=0 \\ &q_{n,2}(x=H/2)=-h_2(\bar{T})\big(T_f(H/2)-T_s(H/2)\big)-\varepsilon_2\sigma\big(T_f^{\,4}(H/2)-T_s^{\,4}(H/2)\big) \end{aligned} \tag{10}$$

In the equations (9) and (10) convection coefficient h_2 on the fire exposed side was 25 W/m^2K and the resultant emissivity ε_2 = 0,9 as given in [8].

NUMERICAL EXAMPLES OF THE THERMAL ANALYSIS

In the following, calculated examples are compared to the results obtained at the tests at NRC, Canada [1, 5, 6, 9]. The following thermal properties were used; thermal conductivity λ_c

(W/m K) of normal weight concrete and specific heat of normal-weight concrete c_c (J/kg K) given in the Eurocode 4 (see Fig. 2a);

$$\lambda_c = 2 - 0.24\ (T_c / 120) + 0.012\ (T_c / 120)^2 \tag{11}$$

$$c_c = 900 + 80(T_c / 120) - 4(T_c / 120)^2 \tag{12}$$

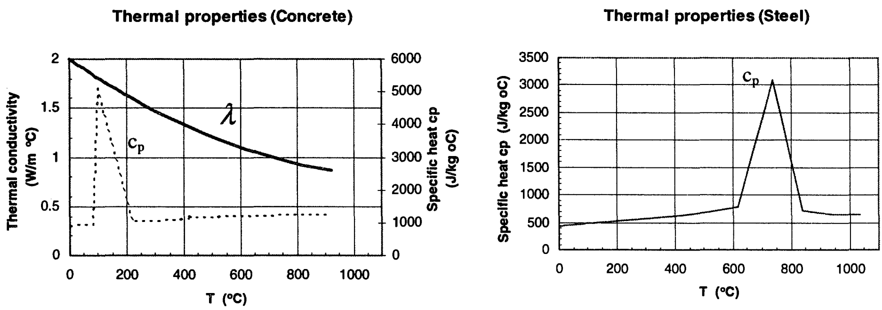
The moisture content w was considered completing the effective specific heat by a triangular situated between T_{c1}=100 °C and T_{c2}=200 °C with peak at 100 °C (see Fig. 2a). The value of the peak was calculated from the equation $c_{c,w\max} = 2\ w\ H_v / (T_{c1} - T_{c2})$ using value H_v= 2.257 x 10^6 J/kg for the heat of vaporization of water and value w = 10 % for the moisture content of concrete. Same moisture content has been used also by Lie and Chabot [8] and Wang [11] for the same test results. Specific heat of steel was calculated using the equations given in Eurocode 4 (Fig. 2b).

The furnace temperature was assumed to follow the same ASTM-E119 standard temperature-time curve as in the tests at NRCC see [1];

$$T_f = 20 + 750\ (1 - \exp(-3.79553\sqrt{\tau}) + 170.41\sqrt{\tau} \tag{13}$$

where T_f is temperature in °C and τ is time in hours. This is almost the same standard temperature-time curve that as given in the standard ISO 834.

Results of the thermal analysis compared to the test results are shown in Figures 3 - 6. A reasonable agreement between the calculated and tested results can be observed. It must be taken into account that Eurocode material properties were used without any fitting. In the computations by Lie and Chabot [8] and Wang [11] similar accuracy has been obtained.



a) **b)**

FIGURE 2. Thermal properties used for a) concrete and b) steel.

The problem in thermal analysis of concrete structures is that moisture movement cannot be taken into account properly without solving coupled heat and moisture transfer problem [15]. In order to get reasonable results a rather high moisture content had to be used, because of the migration of moisture to the center of the column.

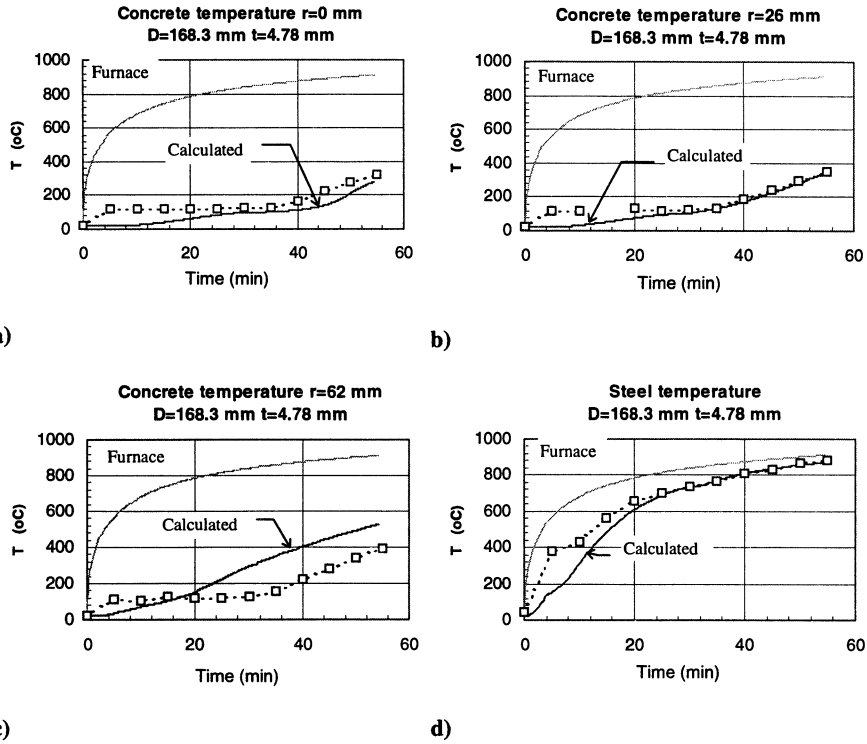


FIGURE 3. Calculated and measured temperatures in the concrete and steel (column diameter 168.3 mm, wall thickness 4.78 mm), (upper line is the standard furnace temperature, thick solid lines are calculated values and dotted lines with boxes are measured values).

CALCULATION OF THE BUCKLING RESISTANCE

Resistance of Composite Column Applying Eurocode 4 Simple Method

According to Chapter 4.3.6.1 of the Eurocode 4 Part 1.2 [4] the resistance of a composite column in concentric axial compression is obtained from the equation

$$N_{fi,Rd} = \chi \, N_{fi,pl.Rd} \tag{14}$$

where χ is the reduction coefficient and depends on the non-dimensional slenderness ratio $\overline{\lambda}_\theta = \sqrt{N_{fi,plR} / N_{fi,cr}}$.

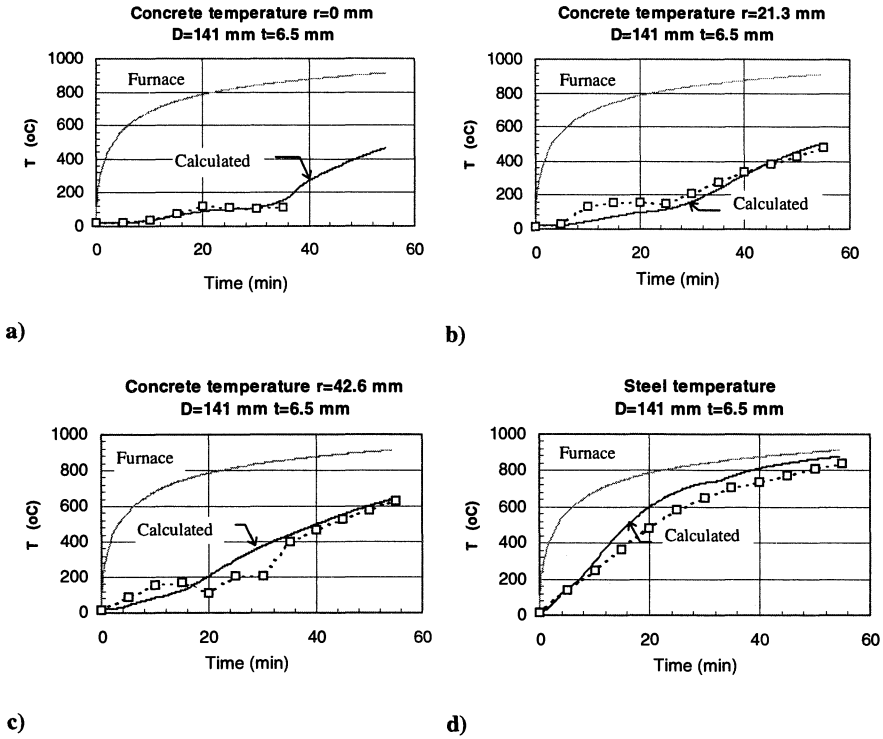


FIGURE 4. Calculated and measured temperatures in the concrete and steel, column diameter 141.3 mm, wall thickness 6.5 mm. (upper solid line is the standard furnace temperature, thick solid lines are calculated values and dotted lines with boxes are measured values). Test data from Lie and Caron [5].

The Euler buckling load in the fire situation is $N_{fi,cr} = \pi^2 \, EI_{fi,eff} / l_\theta^2$, where l_θ is the buckling length of the column in the fire situation. $N_{fi,pl.Rd}$ is the design value of plastic resistance to axial compression in the fire situation. The non-dimensional slenderness ratio is calculated using equation

$$\chi = \frac{1+\alpha(\overline{\lambda}_\theta - 0.2) + \overline{\lambda}_\theta^{\,2}}{2\,\overline{\lambda}_\theta^{\,2}} - \frac{\sqrt{(1+\alpha(\overline{\lambda}_\theta - 0.2) + \overline{\lambda}_\theta^{\,2})^2 - 4\overline{\lambda}_\theta^{\,2}}}{2\,\overline{\lambda}_\theta^{\,2}} \tag{15}$$

where $\alpha = 0.49$ for buckling curve c and $\alpha = 0.21$ for buckling curve a.

According to Eurocode 4 [4] the cross section of a composite column may be divided into various parts concerning the steel profile, concrete and the reinforcing bars. In order to be able to calculate the plastic resistance to axial compression $N_{fi,pl.Rd}$ and effective flexural stiffness $EI_{fi,eff}$ the cross-sectional area of the column is subdivided into a number of circular or square ring elements (Fig. 1a and 1b) that coincide to the FE grid of the temperature analysis.

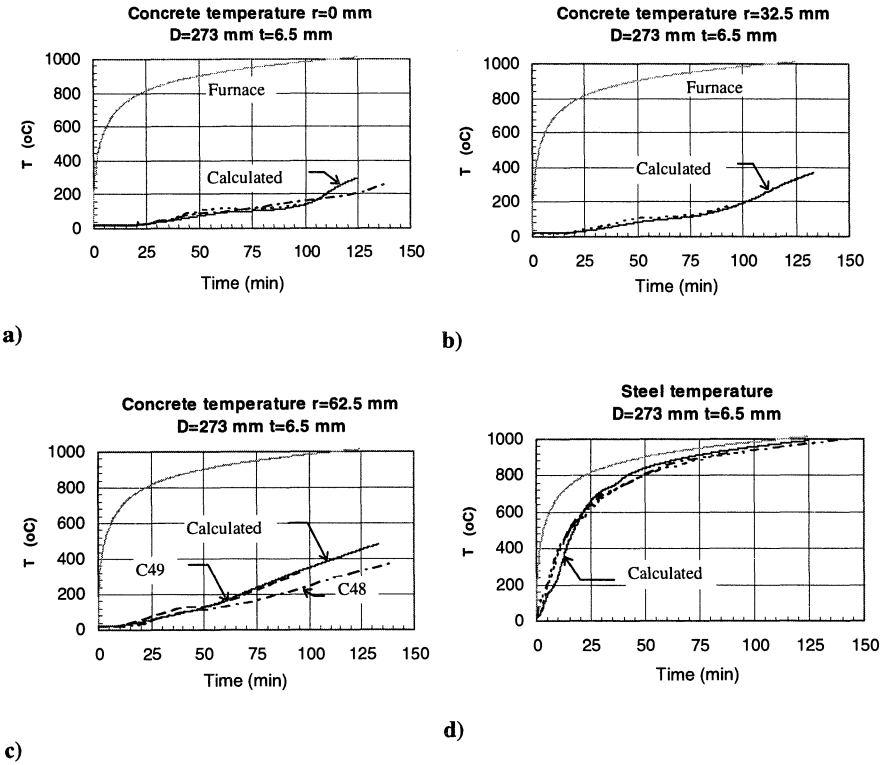


FIGURE 5. Calculated and measured temperatures at various depths in the concrete and steel (upper thin solid line is the standard furnace temperature, thick solid lines are calculated values and dotted lines are measured values). Test data from circular columns C48 and C49 of size 273 mm x 6.3 mm [1].

The plastic resistance to axial compression and effective flexural stiffness of the cross-section of the circular column are calculated using following numerical integration formulas

$$N_{fi,pl.Rd} = \pi \sum_{e} ((r_{2,a}^{e})^2 - (r_{1,a}^{e})^2) f_{a\max}(\overline{T}^{e}) + \sum_{j} (A_s^j f_{s\max}(T_s^j) - A_s^j f_{c\max}(T_s^j)) \qquad (16)$$

$$EI_{fi,eff} = \frac{\pi}{4}\sum_e ((r_{2,a}^e)^4 - (r_{1,a}^e)^4)E_a(\bar{T}^e) + \sum_j (E_s(T_s^j)\ A_s^j y_s^2 - E_c(T_s^j)A_s^j y_s^2) \tag{17}$$

where the first term of equations is summation taken over all ring elements, either concrete or steel. The second term is summation taken over all reinforcement bars located at $y_s = r_s \sin\varphi$. Temperature $\bar{T}^e = (T_1^e + T_2^e)/2$ is calculated at the center of the square ring element.

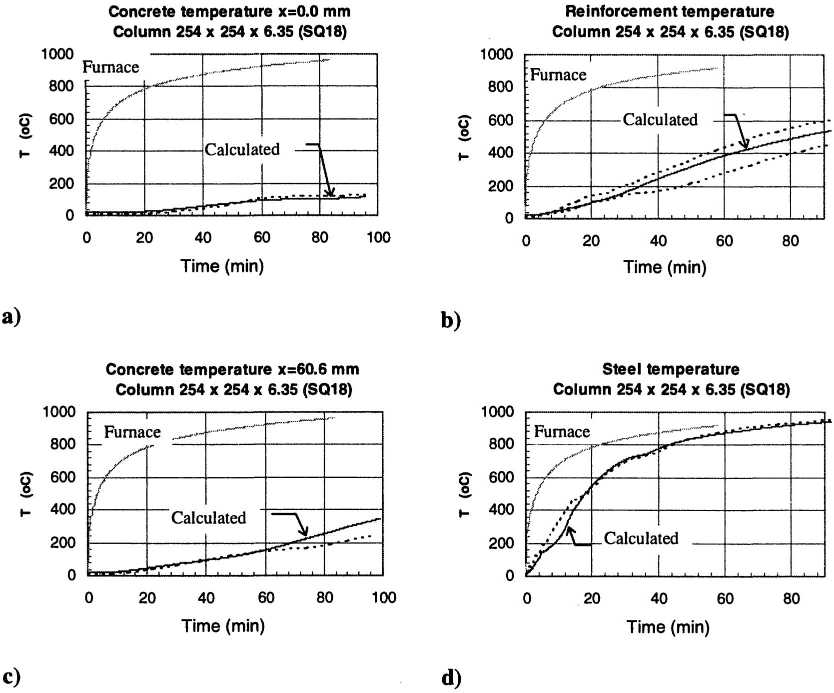


FIGURE 6. Calculated and measured temperatures at various depths in the concrete and steel (upper thin solid line is the standard furnace temperature, thick solid lines are calculated values and dotted lines are measured values). Test data of square column 254 mm x 254 mm x 6.3 mm [1].

The plastic resistance to axial compression and effective flexural stiffness of the cross section of the square column are calculated using following approximate integration formulas

$$N_{fi,pl.Rd} = 4\sum_e ((x_{2,a}^e)^2 - (x_{1,a}^e)^2) f_{a\max}(\bar{T}^e) + \sum_j (A_s^j\ f_{s\max}(T_s^j) - A_s^j\ f_{c\max}(T_s^j)) \tag{18}$$

$$EI_{fi,eff} = \frac{4}{3}\sum_e ((x_{2,a}^e)^4 - (x_{1,a}^e)^4)E_a(\bar{T}^e) + \sum_j (E_s(T_s^j)A_s^j x_s^2 - E_c(T_s^j)A_s^j x_s^2) \tag{19}$$

Note that there is no reinforcement in the thermal analysis. The temperature of each reinforcement bar T_s is here assumed to be the same as the concrete temperature at that spatial location where the reinforcement bar is situated.

TABLE 1. Circular column data used in the computation and calculated resistances (c-curve) compared to the tests results from Lie and Caron [5] and iBMB [13].

Column id. n:o as in [5, 13]	Section diameter (mm)	Steel thickness (mm)	Test Load (kN) N_{test}	Fire resistance (min) in test	Steel yield stress f_y (MPa)	Cylinder strength fc (MPa) at test date	$\bar{\lambda}_\theta$	Calc. resistance $N_{fi,Rd}$ (kN)	Calculated resistance/ Test load $N_{fi,Rd}$ / N_{test}
1 [5]	141.3	6.55	131	57	433	31.0	0.912	126	0.96
2 [5]	168.3	4.78	218	56	374	35.4	0.952	227	1.04
3 [5]	219.1	4.78	492	80	347	31.0	0.779	387	0.79
4 [5]	219	5.56	384	102	347	32.3	0.792	306	0.79
5 [5]	219	8.2	525	82	396	31.7	0.686	426	0.81
6 [5]	273	5.6	574	112	445	28.6	0.661	613	1.07
7 [5]	273	5.6	525	133	445	29.0	0.675	516	0.98
8 [5]	273	5.6	1000	70	445	27.0	0.596	823	0.82
10 [5]	324	6.4	1050	93	474	24.0	0.511	1073	1.02
11 [5]	356	6.4	1050	111	446	23.8	0.480	1247	1.19
13 [5]	406	12.7	1900	71	399	27.6	0.349	2519	1.32
S3[13]	219	6.3	380	71	428	40.	1.018	428	1.12
								mean	**0.99**

Numerical Examples of the Resistance Calculations

The resistances of composite column in axial compression $N_{fi,Rd}$ were obtained using the Eurocode 4 method in the way described in this paper. The mechanical properties given in Eurocode 4 Chapter 3.2 for both steel and concrete were applied. The modulus of elasticity of concrete is not explicitly given in the Eurocode 4. Here the tangential modulus deduced from the stress-strain model of concrete by derivation with respect to $\varepsilon_{c,\theta}$ was used;

$$\sigma_{c,\theta} = f_{c,\theta}\left[3\left(\frac{\varepsilon_{c,\theta}}{\varepsilon_{cu,\theta}}\right) / \left\{2+\frac{\varepsilon_{c,\theta}}{\varepsilon_{cu,\theta}}\right\}^3\right]$$
$$E_{c,\theta} = \frac{\partial\sigma_{c,\theta}}{\partial\varepsilon_{c,\theta}} = \frac{3}{2}\frac{f_{c,\theta}}{\varepsilon_{cu,\theta}} \tag{20}$$

The measured values of yield strength of the steel and concrete (at the date of the test if known) were used. In the NRCC [1, 5] fire resistance tests 3,81 m long columns were tested both ends clamped and buckling length of 2 m recommended by Lie and Chabot [8] was used in the calculations. In NRC test [9] the column was hinged at both ends and buckling length 3,8 m was used. The iBMB test was conducted one end clamped and the other end hinged and buckling length of 2,62 m of the test report [13] was used. Applied strength values and calculated results using c-curve are shown in the Table 1 and 2 and compared to the test results. The resistance of circular non-reinforced columns can be predicted rather well, the mean of the ratios between the calculated and tested result being 0,99. The results with the square columns are less satisfactory. Mean of the ratios between the calculated and tested result is 0,73 which is on the safe side. Especially the results with smaller tubes are very much on the safe side. One reason for this may be that the temperature calculation method overestimates the temperature of the composite column at corners where the reinforcement is located.

TABLE 2. Dimensions of square columns, test loads, fire resistance times, stress values. Test data from Chabot and Lie [1] and Myllymäki, Lie and Chabot [9].

Column id. n:o as in [1, 9]	Section height (mm)	Steel thickness (mm)	Test load (kN)	Fire resistance (min) in test	Steel and reinf. yield stress f_y/f_s (MPa)	Cylinder strength fc (MPa) at test date	$\bar{\lambda}_\theta$	Calc. capacity $N_{fi,Rd}$ (kN)	Calculated /Test load $N_{fi,Rd} / N_{test}$
SQ-3 [9]	150	5	140	83	418/596	37.8	1.571	84	0.60
SQ-12 [1]	203	6.3	500	150	350/400	47	0.598	264	0.53
SQ-13 [1]	203	6.3	930	105	350/400	47	0.652	510	0.55
SQ-18 [1]	254	6.3	1440	113	350/400	48.1	0.549	1210	0.84
SQ-19 [1]	254	6.3	2200	70	350/400	48.1	0.488	1891	0.86
SQ-22 [1]	305	6.3	3400	39	350/400	47	0.364	4088	1.20
								mean	**0.76**

CONCLUSIONS

A simplified numerical method to predict the fire resistance of concrete filled tubular composite columns has been presented. The temperature analysis of the composite column was formulated as a one dimensional Galerkin FE-solution of the heat conduction equation. In the analysis both axisymmetric elements in the cylindrical case and one dimensional elements in a square case were applied. In the former case solution is exact but in the latter case the solution is an approximation for design purposes. The simple design method of the Eurocode 4 has been applied in the calculation of the load-bearing capacity of the tubular composite columns. Both the temperature analysis and structural analysis were conducted with a code written in a spreadsheet program Microsoft Excel. This made it simple and efficient design tool. Calculated temperatures and capacities were compared to the test results of concentrically loaded circular and square tubular columns filled with reinforced and non-reinforced concrete. Results of the cylindrical columns were in satisfactory agreement with the test results. Calculated load-bearing capacities of the small square tubes filled with reinforced concrete were too conservative. This may be due to the approximate nature of the temperature analysis especially when we consider temperature of reinforcement in the corners of the section.

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